

*June 1998*

# ***Mathematics 30***

## ***Grade 12 Diploma Examination***

### ***Description***

Time: 2.5 h. You may take an additional 0.5 h to complete the examination.

This is a **closed-book** examination consisting of

- 40 multiple-choice questions and 9 numerical-response questions, of equal value, worth 70% of the examination
- 3 written-response questions, of equal value, worth 30% of the examination

A tear-out formula sheet and a z-score page are included in this booklet.

All graphs on this examination are computer-generated.

### ***Instructions***

- Consider all numbers used in the questions to be **exact** numbers and not the result of a measurement.
- Fill in the information required on the answer sheet and the examination booklet as directed by the presiding examiner.
- You are expected to provide your own scientific calculator.
- Carefully read the instructions for each part before proceeding.
- The presiding examiner will collect your answer sheet and examination booklet and send them to Alberta Education.
- Do not fold the answer sheet.

***Note:*** The perforated pages at the back of this booklet may be torn out and used for your rough work.  
***No marks*** will be given for work done on the tear-out pages.

### Multiple Choice

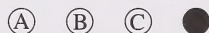
- Read each question carefully and decide which of the choices completes the statement or answers the question.
- Locate that question number on the separate answer sheet provided and fill in the circle that corresponds to your choice.

### Example

This diploma examination is for  
the subject of

- A.** biology  
**B.** physics  
**C.** chemistry  
**D.** mathematics

## Answer Sheet



- Use an **HB pencil only**.
- If you wish to change an answer, erase **all** traces of your first answer.

### Numerical Response

- Read each question carefully.
- Record your answer on the answer sheet provided by writing it in the boxes and then filling in the corresponding circles.
- **Enter the first digit of your answer in the left-hand box and leave any unused boxes blank.**
- **Use an HB pencil only.**
- If you wish to change an answer, erase **all** traces of your first answer.

### Sample Questions and Solutions

### Example

Correct to the nearest tenth of a radian,  $40^\circ$  is equal to \_\_\_\_\_ rad.

$$40^\circ = 0.6981317008 \dots \text{rad}$$

**Record 0.7 on the answer sheet —**

### Example

For the arithmetic series  $-8 + (-5) + (-2) + \dots + (85)$ , the number of terms is \_\_\_\_\_.

$$85 = -8 + (n - 1)(3)$$

$$93 = 3n - 3$$

$$n = 32$$

Record 32 on the answer sheet

→ 

3	2		
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•	•		
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

### Example

Place the following diploma examination subjects in alphabetical order.

- 1 biology
- 2 physics
- 3 chemistry
- 4 mathematics

(Record your answer as 

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.)

Answer: 1342

Record 1342 on the answer sheet

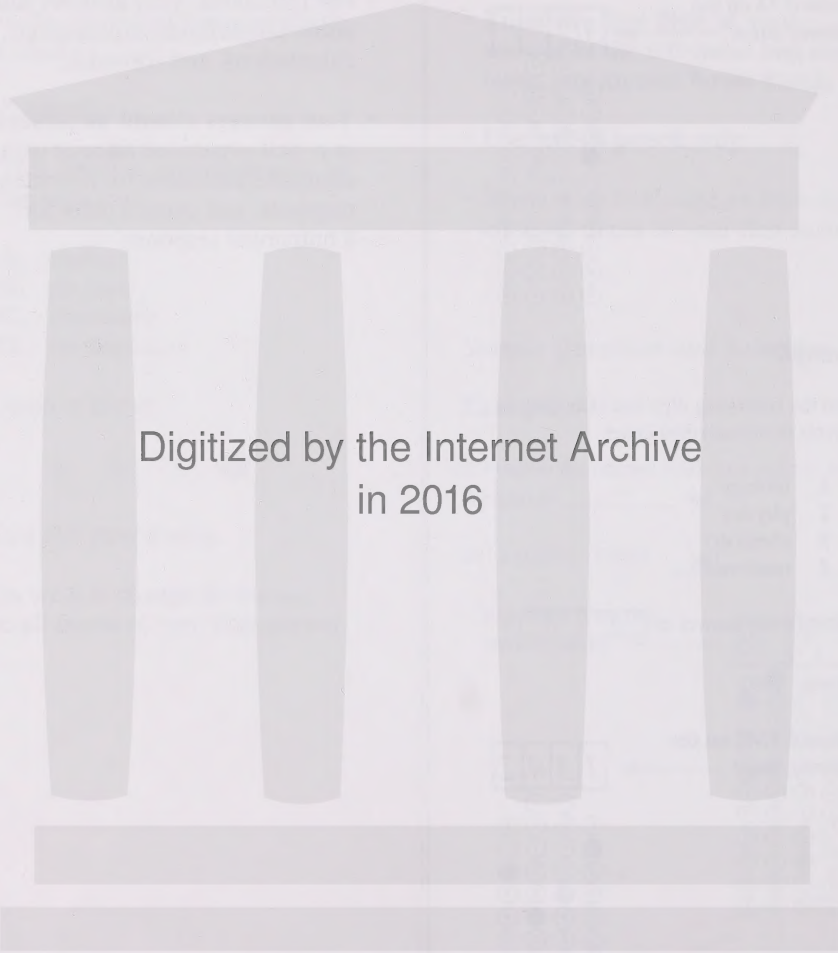
→ 

1	3	4	2
---	---	---	---

•	•		
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

### Written Response

- Read each question carefully.
- Write your answers in the examination booklet as neatly as possible.
- For full marks, your answers **must show all** pertinent explanations, calculations, and formulas.
- Your answers **should be** presented in a well-organized manner using complete sentences for a written response, and correct units for a numerical response.



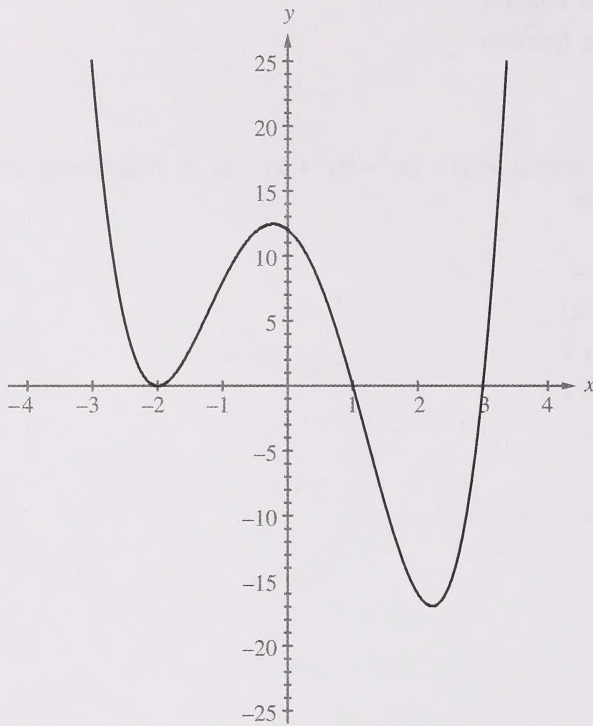
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1. The equation  $P(x) = x^3 + 2x^2 + x + 4$  is an example of a
- A. quadratic function
  - B. quartic function
  - C. linear function
  - D. cubic function
2. If the polynomial  $P(x) = 3x^3 - 9x^2 + kx - 12$  is divisible by  $x - 3$ , then it is also divisible by
- A.  $3x - 4$
  - B.  $3x + 4$
  - C.  $3x^2 + 4$
  - D.  $3x^2 - 4$

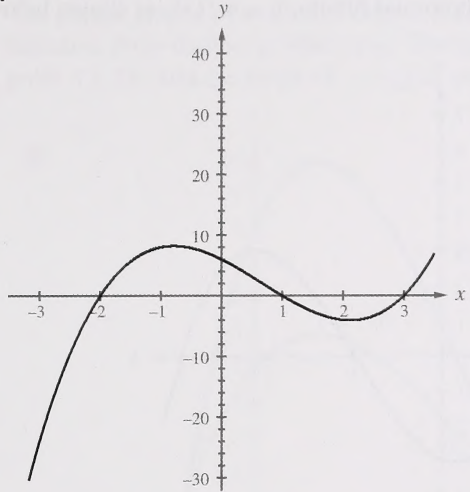
Use the following information to answer the next question.

The polynomial function  $P$  has only integral zeros. The graph of  $y = P(x)$  is shown below.

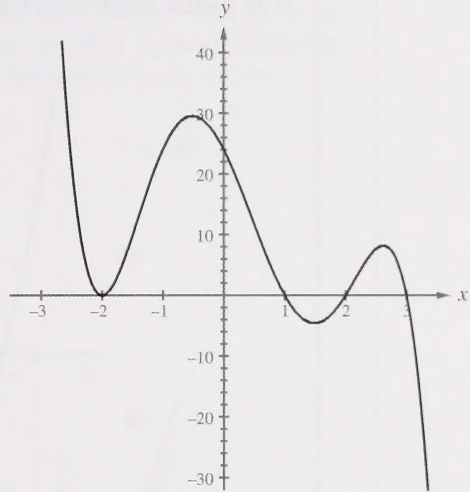


3. The graph of  $y = (x + 2) P(x)$  is

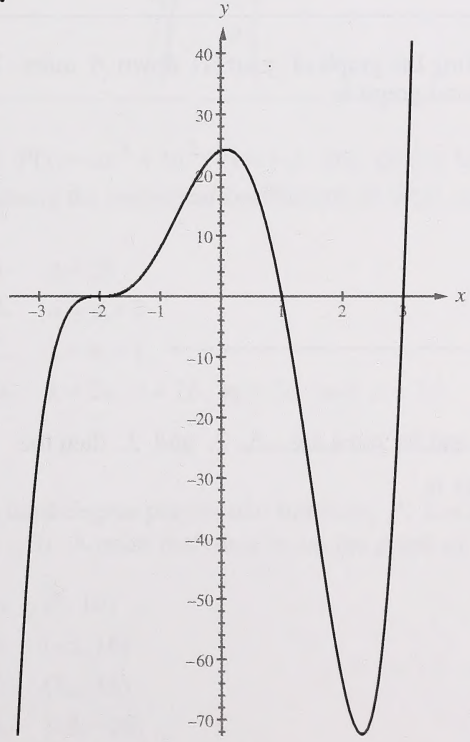
A.



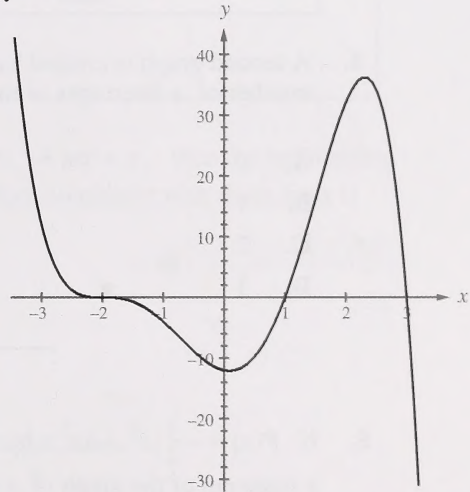
B.



C.

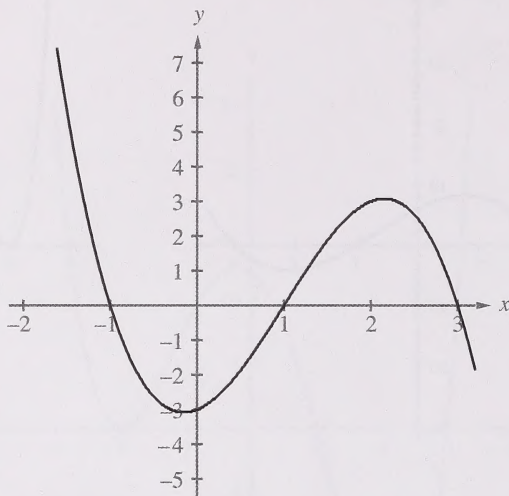


D.



Use the following information to answer the next question.

The partial graph of a cubic polynomial function,  $y = f(x)$ , is shown below.



4. A second graph is created by shifting the graph of  $y = f(x)$  down 6 units. The number of  $x$ -intercepts of the second graph is

A. 0  
B. 1  
C. 2  
D. 3

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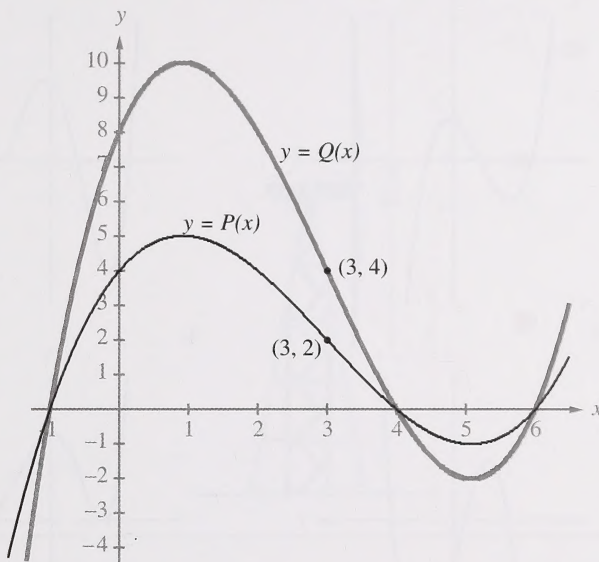
5. If  $P(x) = -\frac{1}{2}(x^3 + ax^2 + bx + c)$  and its zeros are  $-5$ ,  $2$ , and  $3$ , then the  $y$ -intercept of the graph of  $y = P(x)$  is

A.  $-30$   
B.  $-15$   
C.  $15$   
D.  $30$



Use the following information to answer the next question.

The partial graphs of two third-degree polynomial functions,  $P$  and  $Q$ , have the same three distinct  $x$ -intercepts. The graph of  $y = P(x)$  passes through the point  $(3, 2)$ , and the graph of  $y = Q(x)$  passes through the point  $(3, 4)$ .



6. If  $P(x) = ax^3 + bx^2 + cx + d$  and  $Q(x) = kx^3 + lx^2 + mx + n$ , then the relationship among the numerical coefficients of  $P(x)$  and  $Q(x)$  consistent with these facts is

- A.  $a = 2k$
- B.  $n = d + 2$
- C.  $k = a + 2$
- D.  $k = 2a$ ,  $l = 2b$ ,  $m = 2c$ , and  $n = 2d$

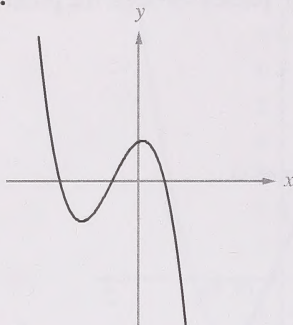
7. A third-degree polynomial function,  $P$ , has a remainder of 16 when divided by  $(x - 3)$ . A point that must be on the graph of  $y = P(x)$  is

- A.  $(3, 16)$
- B.  $(-3, 16)$
- C.  $(3, -16)$
- D.  $(-3, -16)$

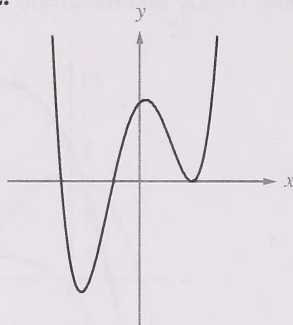
Use the following information to answer the next question.

The graphs of four polynomial functions are shown below. Each polynomial function is of the minimum degree consistent with its graph.

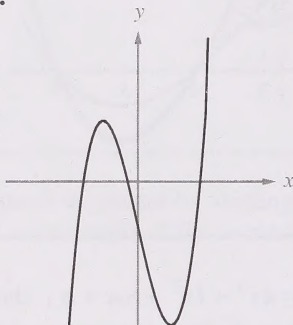
1.



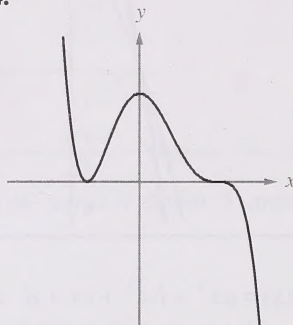
2.



3.



4.



### Numerical Response

1. Match each of the polynomial graphs shown above with a description that applies to it.

A graph of a fourth-degree polynomial function with a zero of multiplicity 2 \_\_\_\_  
(Record in column 1 of numerical response 1)

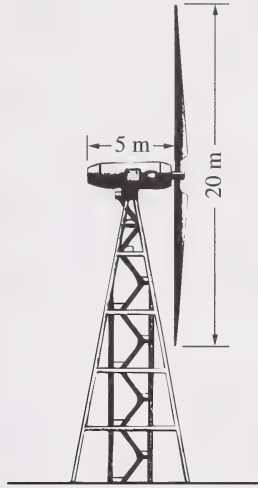
A graph with a negative y-intercept \_\_\_\_  
(Record in column 2 of numerical response 1)

A graph of a polynomial function with a negative coefficient of the term of the largest degree and all zeros of multiplicity 1 \_\_\_\_  
(Record in column 3 of numerical response 1)

A graph of a fifth-degree polynomial function \_\_\_\_  
(Record in column 4 of numerical response 1)

Use the following information to answer the next question.

Wind turbines can be used to produce electricity. In the diagram below, the wind turbine has a fibreglass blade 20 m in length that is attached to a generator 5 m in length.



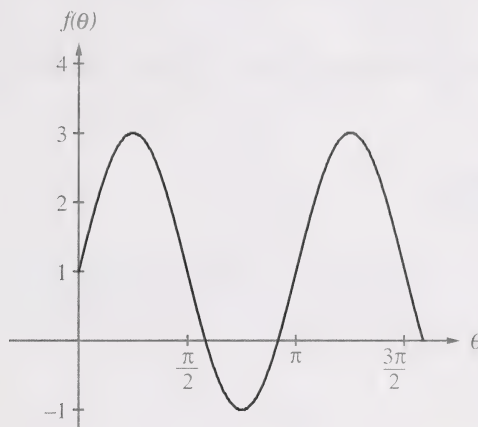
8. The arc length traversed by a tip of the blade during a rotation of  $\frac{\pi}{4}$  radians is
- A.  $\frac{5\pi}{8}$  m
- B.  $\frac{5\pi}{4}$  m
- C.  $\frac{5\pi}{2}$  m
- D.  $5\pi$  m

9. The amplitude of a given sine function,  $f(\theta) = a \sin \theta + d$ , is 6. If the maximum value of  $f$  is 8, then the minimum value of  $f$  is
- A. -8  
B. -6  
C. -4  
D. 2
10. If  $\cot \theta = -\frac{3}{4}$  and  $\csc \theta < 0$ , then  $\sin^2 \theta - \cos^2 \theta$  is equal to
- A. -1  
B.  $-\frac{7}{25}$   
C.  $\frac{7}{25}$   
D. 1
11. The complete set of solutions for  $6 \sin(2\theta) - 3 = 0$ ,  $0 \leq \theta < 2\pi$ , is
- A.  $\frac{\pi}{3}, \frac{5\pi}{3}$   
B.  $\frac{\pi}{6}, \frac{5\pi}{6}$   
C.  $\frac{\pi}{6}, \frac{\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}$   
D.  $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$



Use the following information to answer the next question.

A partial graph of  $f(\theta) = 2 \sin 2\theta + 1$  is shown below.



12. The minimum period of  $f(\theta)$  is

- A.  $\frac{\pi}{2}$
  - B.  $\pi$
  - C.  $2\pi$
  - D.  $4\pi$
- 

13. The y-intercept of the graph of  $y = a \sin \theta + d$  is

- A.  $d$
- B.  $-d$
- C.  $a + d$
- D.  $a - d$

**Numerical Response**

2. If  $\theta = 1.8$  radians, then correct to the nearest hundredth, the value of  $\sin^2 \theta$  is \_\_\_\_\_.

(Record your answer in the numerical-response section of the answer sheet.)

**Numerical Response**

3. Evaluate each of the following four trigonometric expressions. Identify the order of the values from smallest to largest. Record this order by listing the number of each expression, starting in the extreme left column of numerical response 3.

1  $\tan^2 \theta - \sec^2 \theta$

2  $\tan \theta - \frac{\sin \theta}{\cos \theta}$

3  $5 \sin^2 \theta + 5 \cos^2 \theta$

4  $\frac{1}{7} \sin \theta \csc \theta$

Answer: \_\_\_\_\_

(Record your answer in the numerical-response section of the answer sheet.)

14. The expression  $\log_a \left( \frac{y^3 \sqrt{x}}{5} \right)$  is equivalent to

A.  $\log_a (3y) + \log_a \left( \frac{1}{2} x \right) - \log_a 5$

B.  $\log_a (3y) + 2 \log_a x - \log_a 5$

C.  $3 \log_a y + \frac{1}{2} \log_a x - \log_a 5$

D.  $3 \log_a y + 2 \log_a x - \log_a 5$

15. John invested  $P$  dollars at 3% compounded annually for a 20-year period. The amount of money that John will have at the end of each year is increased from the amount at the end of the previous year by a factor of
- A. 0.03
  - B.  $P$
  - C. 1.03
  - D.  $1.03P$
16. If  $\log_x\left(\frac{1}{9}\right) = -2$ , then the value of  $x$  is
- A. 3
  - B.  $-3$
  - C.  $\frac{1}{3}$
  - D.  $-\frac{1}{3}$
17. If  $\log x = 2.4$  and  $\log y = -1.6$ , then the value of  $\frac{x}{y}$  is
- A.  $-1.5$
  - B. 4
  - C.  $10^{-1.5}$
  - D.  $10^4$

18. The number of E. coli bacteria at time  $t$  in hours is given by  $N(t) = N_0(8)^t$ , where  $N_0$  is the initial number of bacteria at  $t = 0$ . If the initial number of bacteria is 4 000, then the expected number of bacteria 1 h 15 min later is
- A. 4 600  
B. 5 000  
C. 43 713  
D. 53 817
19. A student wishes to graph  $y = \log_3 x$  on a calculator, but her calculator can only graph logarithmic functions if the equations are expressed in common logarithms. She could obtain the graph of  $y = \log_3 x$  by graphing
- A.  $y = \frac{\log x}{3}$   
B.  $y = \frac{\log x}{\log 3}$   
C.  $y = \log(x - 3)$   
D.  $y = \log\left(\frac{x}{3}\right)$



Use the following information to answer the next question.

A beam of light passes through a piece of opaque material. The intensity  $I$  of the beam of light as it exits the material is given by

$$I = (4^{-ct})I_0,$$

where  $I_0$  is the initial intensity,  $c$  is the absorption factor, and  $t$  is the thickness of the material in centimetres.

20. If the intensity of the beam  $I$ , the initial intensity  $I_0$ , and the absorption factor  $c$  are known and  $\frac{I}{I_0} = R$ , then an expression for  $t$  is

A.  $\frac{-\log R}{4c}$

B.  $\frac{-\log R}{c \log 4}$

C.  $\frac{-1}{c} \log\left(\frac{R}{4}\right)$

D.  $\frac{-\log(R - 4)}{c}$

### Numerical Response

4. Given the equation  $\log_9 x = \log_4 64$ , the value of  $x$ , to the nearest integer, is \_\_\_\_\_.

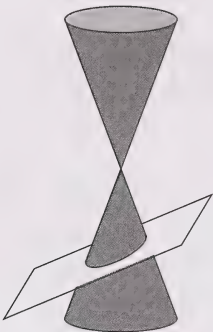
(Record your answer in the numerical-response section of the answer sheet.)

21. The diagram that **best** illustrates how a plane intersects a double-napped cone to form a non-degenerate hyperbola is shown in

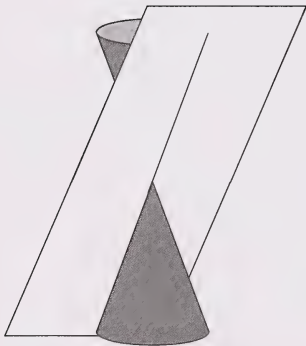
A.



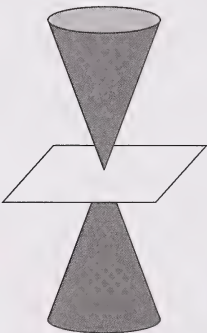
B.



C.

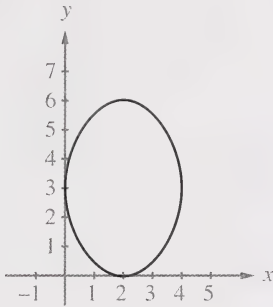


D.

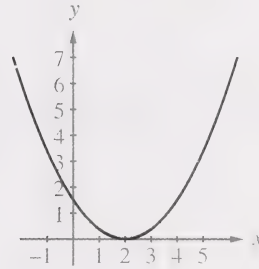


Use the following information to answer the next question.

Brenda and Paul were each given an assignment to graph the quadratic relation  $Ax^2 + Cy^2 + Dx + Ey + F = 0$  for given values of  $A$ ,  $C$ ,  $D$ ,  $E$ , and  $F$ . Brenda entered the relation correctly, obtaining the graph below on the left. Paul accidentally left out one term and obtained the graph below on the right.



**Brenda's Graph**

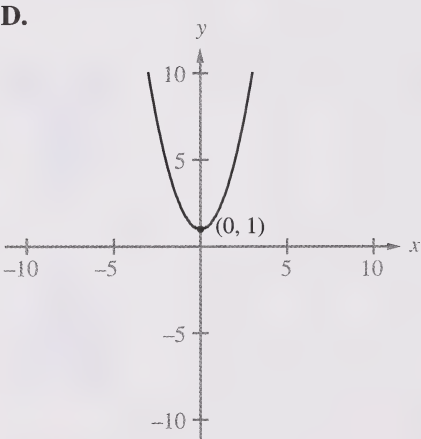
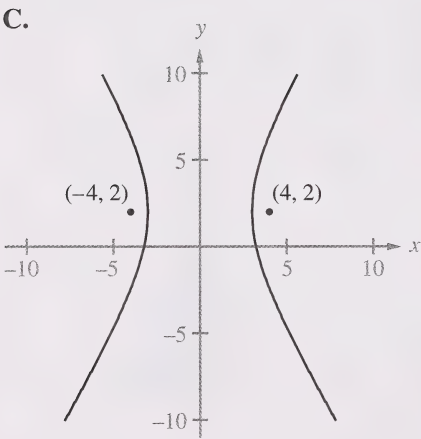
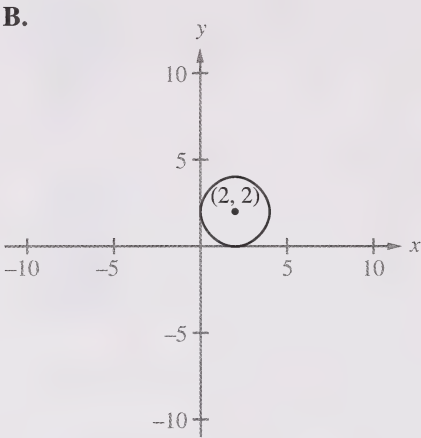
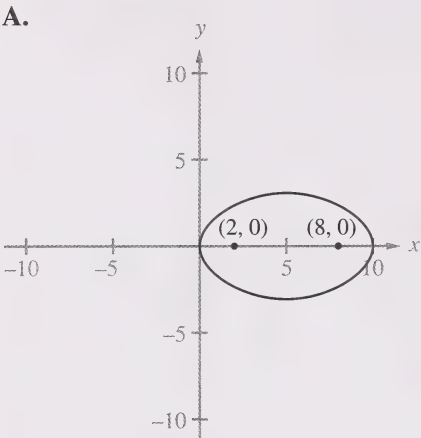


**Paul's Graph**

22. The term that was left out by Paul but not by Brenda was

- A.  $Ax^2$
- B.  $Cy^2$
- C.  $Dx$
- D.  $Ey$

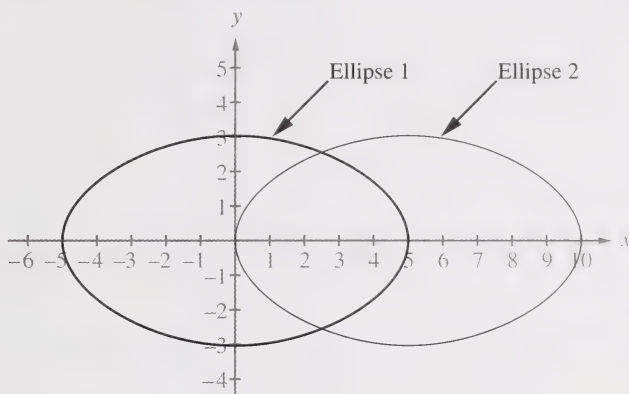
23. A quadratic relation is given by the equation  $Ax^2 + Cy^2 + Dx + Ey + F = 0$ , where  $A \times C \times D \times E \neq 0$ . The foci of the ellipse and hyperbola, the centre of the circle, and the vertex of the parabola are shown. Which of the following graphs could **not** be the graph of this quadratic relation?





Use the following information to answer the next question.

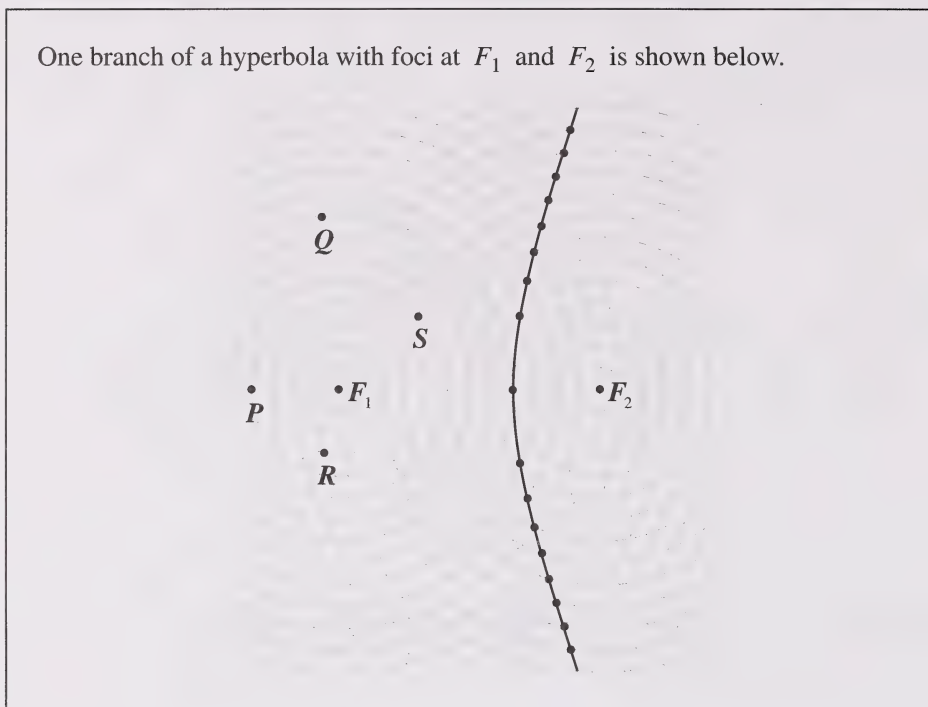
Two ellipses with identical shapes are shown below. The graph of ellipse 2 is obtained by moving the graph of ellipse 1 five units to the right.



24. Which of the following statements about the ellipses is **true**?
- A. The values of the eccentricities of the ellipses are different.
  - B. The general equation of each ellipse is  $Ax^2 + Cy^2 + Dx + Ey + F = 0$ , where the values for parameters  $A$ ,  $C$ , and  $F$  are the same.
  - C. The foci are closer together in the first ellipse than in the second.
  - D. The sum of the distances from a point on an ellipse to its foci is the same for each ellipse.

Use the following information to answer the next question.

One branch of a hyperbola with foci at  $F_1$  and  $F_2$  is shown below.



25. A point that could lie on the other branch of the hyperbola is labelled

- A.  $S$
- B.  $Q$
- C.  $R$
- D.  $P$

26. The equation  $\frac{\sqrt{x^2 + (y - 5)^2}}{\sqrt{(y - 2)^2}} = \frac{a}{b}$ , where  $0 < \frac{a}{b} < 1$ , defines a family of ellipses with a focus at
- A.  $(0, 2)$
  - B.  $(0, -2)$
  - C.  $(0, 5)$
  - D.  $(0, -5)$

### Numerical Response

5. A parabola with a vertex at  $(1, 2)$  and a focus at  $(c, 2)$  has a directrix defined by  $x = -15$ . The value of  $c$ , to the nearest tenth, is \_\_\_\_\_.

(Record your answer in the numerical-response section of the answer sheet.)

27. If the recursive formula for a sequence is

$$\begin{cases} t_1 = m \\ t_n = 3t_{n-1} - 2, \quad n \geq 2, \quad n \in N, \end{cases}$$

then the third term,  $t_3$ , is

- A.  $3m - 2$
- B.  $9m - 8$
- C.  $18m - 20$
- D.  $27m - 26$

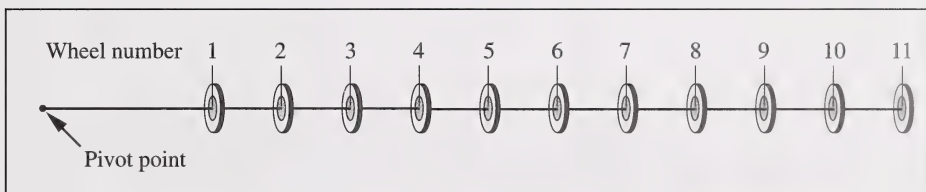
28. The sum of the first 10 terms of the arithmetic sequence  $\sin \theta, 2 \sin \theta, 3 \sin \theta, \dots, 10 \sin \theta$  is 44. The value of the common difference is
- A. 0.20
  - B. 0.73
  - C. 0.80
  - D. 0.88
29. The sum of the terms of a sequence is represented by  $S_n = n^2 + 4n$ ,  $n \geq 1$ ,  $n \in N$ . The general term of the sequence is
- A.  $t_n = 5\left(\frac{8}{5}\right)^{n-1}$
  - B.  $t_n = 5(3)^{n-1}$
  - C.  $t_n = 3n + 2$
  - D.  $t_n = 2n + 3$
30. The points  $P(1, k)$ ,  $Q(2, 4)$ , and  $R(3, 2)$  all lie on a straight line. The y-coordinates of points  $P$ ,  $Q$ , and  $R$  form a three-term sequence. If  $t_1 = k$ ,  $t_2 = 4$ , and  $t_3 = 2$ , then the value of  $k$  and the type of sequence are, respectively,
- A. 6 and arithmetic
  - B. 8 and arithmetic
  - C. 6 and geometric
  - D. 8 and geometric



31. A geometric series with the common ratio  $\sqrt{3}$  is described by

- A.  $\sum_{n=1}^k (\sqrt{3})^n$
- B.  $\sum_{n=1}^k (3 + \sqrt{3})^n$
- C.  $\sum_{n=1}^k \sqrt{3}n$
- D.  $\sum_{n=1}^k (3 + \sqrt{3})n$

*Use the following information to answer the next question.*



32. Some farmers irrigate their fields using a system where water is pumped along a pipe. The pipe is supported and driven by wheels. The first wheel is attached 50 m from a pivot point, and all the other wheels are attached at 20 m intervals along the pipe. These wheels move in concentric circles centred at the pivot point. Correct to the nearest metre, the **circumference** of the circle traversed by wheel 11 is
- A. 1 571 m
- B. 1 650 m
- C. 1 696 m
- D. 1 920 m

## Numerical Response

6. The sum of the first 30 terms of an arithmetic series is 4 425. If the last term is 220, then the value of the first term, correct to the nearest tenth, is \_\_\_\_\_.

(Record your answer in the numerical-response section of the answer sheet.)

33. A girl named **Kristi Whittall** finds that her name is often misspelled. Her first name has been spelled as Kristi, Christi, Kristy, and Christy. Her last name has been spelled as Whittall, Whittle, Whittall, Wittal, and Whidall. If you assume that these represent all of the spelling variations of her first and last names, then the total number of spelling variations, correct and incorrect, of her full name is
- A. 7  
B. 9  
C. 12  
D. 20
34. What is the number of distinguishable arrangements of the letters in the word **SYLLABLE** if all of the letters must be used in each arrangement?
- A.  $\frac{8!}{3!}$   
B.  $8!$   
C.  $6!$   
D.  $8! - 3!$

35. In how many ways can a gymnastics class of 15 gymnasts be divided into 3 groups so that 3 gymnasts are in one group, 5 gymnasts are in a second group, and 7 gymnasts are in a third group?
- A. 105  
B. 2 184  
C. 360 360  
D. 3 628 800
36. You want to call a friend but have forgotten part of his seven-digit phone number. You know that the first three digits are 555. What is the probability that you will dial the correct phone number on the first try?
- A.  $\frac{21}{1\,000}$   
B.  $\frac{1}{10\,000}$   
C.  $\frac{63}{125}$   
D.  $\frac{1}{3\,628\,800}$
37. The binomial expansion of  $(2x - 3)^{2n-1}$  has 16 terms. The value of  $n$  is
- A. 8  
B. 9  
C. 15  
D. 16

### Numerical Response

7. Pat's parents asked him to complete the following seven chores:

- take out the garbage
- clean his bedroom
- vacuum the living room
- drive his brother to a piano lesson
- wash the car
- do the dishes
- clean the bathroom

They asked Pat to drive his brother to a piano lesson first and then to complete the **rest** of the chores in any order. In how many different orders could Pat complete the chores?

Answer: \_\_\_\_\_

(Record your answer in the numerical-response section of the answer sheet.)

### Numerical Response

8. The number of distinguishable ways that 9 glasses of water can be arranged around the outside of a round table, if only 3 of the glasses are identical, is \_\_\_\_\_.

(Record your answer in the numerical-response section of the answer sheet.)

38. A measure of dispersion of a set of data is the

- A. mean
- B. median
- C. mode
- D. standard deviation

39. Your mark of 80 on a standardized test is higher than the marks of 97% of the students who took the test. If the marks are normally distributed with a mean of 62, then the approximate standard deviation is
- A. 6.0
  - B. 9.6
  - C. 12.0
  - D. 19.2
40. The number of male students attending a college is 1 290. Their ages are normally distributed, and their mean age is 22 years with a standard deviation of 1.4 years. If students over the age of 25 years are classified as mature students, then the approximate number of male students who are classified as mature students is
- A. 21
  - B. 28
  - C. 63
  - D. 83

### Numerical Response

9. For a standard normal distribution, the probability of getting a  $z$ -value between  $z = 1.72$  and  $z = 1.98$ , correct to the nearest hundredth, is \_\_\_\_\_.

(Record your answer in the numerical-response section of the answer sheet.)

*The written-response questions follow on the next page.*



*Use the following information to answer the next question.*

A student is required to sketch the two ellipses represented by Case 1 and Case 2 below.

**Case 1** The value of the eccentricity of an ellipse is  $\frac{2}{3}$ , and the distance between the directrix and the focus is 5 units.

**Case 2** For an ellipse, the distance between  $F_1$  and  $F_2$  is 12 units. The sum of the distances from a point,  $P$ , on the ellipse to two fixed points,  $F_1$  and  $F_2$ , is 14 units.

**Written Response — 10%**

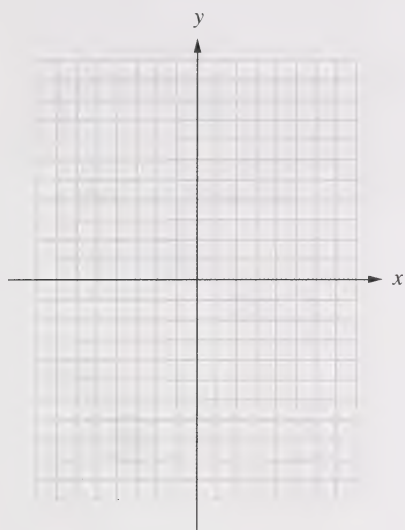
**1.** Three types of graph paper are commonly used when students are studying quadratic relations. An example of each type of graph paper is shown on the next page.

- Identify which of the three types of graph paper would be most appropriate to use to draw the ellipse in Case 1 and which would be most appropriate to use to draw the ellipse in Case 2.
- Clearly describe the steps that you would take to sketch the ellipse for **either Case 1 or Case 2**.

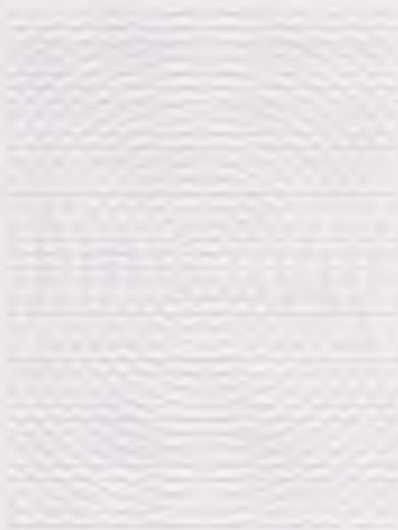
*Written-response question 1 continues on the next page.*



**Graph Paper 1**



**Graph Paper 2**



**Graph Paper 3**



*Written-response question 1 continues on the next page.*

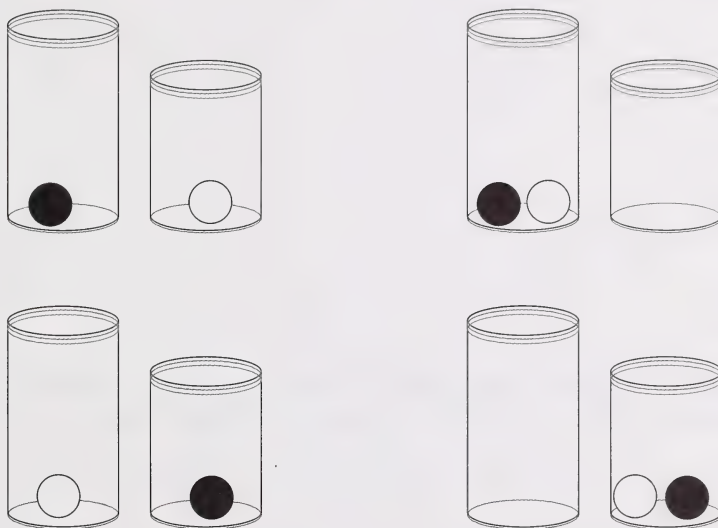
*Written-response question 2 begins on the next page.*



Use the following information to answer the next question.

The number of ways that you can place distinguishable balls in two jars of different sizes is a function of the number of balls you use. The function is  $f(n) = 2^n$ ,  $n \in \mathbb{N}$ , where  $n$  is the number of balls.

Shown in the diagram below are all the ways of placing a black ball and a white ball in two jars of different sizes.



Assume that the balls will always fit in the jars.

**Written Response — 10%**

2. • Complete the chart below so that it shows the relation between any number of distinguishable balls from 1 through 5 inclusive, and the number of ways that the balls can be placed in two jars of different sizes.

$n$	1	2	3	4	5
$f(n)$					

Written-response question 2 continues on the next page.

- Evaluate  $\sum_{k=1}^{12} f(k)$ .

- Another expression representing the number of ways 4 different coloured balls can be placed in two jars of different sizes is

$$({}_4C_4 \times {}_0C_0) + ({}_4C_3 \times {}_1C_1) + ({}_4C_2 \times {}_2C_2) + ({}_4C_1 \times {}_3C_3) + ({}_4C_0 \times {}_4C_4).$$

Explain what the term  $({}_4C_2 \times {}_2C_2)$  represents in the context of this problem.

*Written-response question 2 continues on the next page.*

- The two jars must be distinguishable, for example different sizes, in order for the function  $f(n) = 2^n$ ,  $n \in \mathbb{N}$  to represent the number of ways  $n$  distinguishable balls can be placed in the two jars.

Determine a function,  $g$ , that represents the number of ways  $n$  distinguishable balls can be placed in two identical jars. Justify your answer.

*Written-response question 3 begins on the next page.*

Use the following information to answer the next question.

The following tables were correctly completed by two students. The sequence  $t_n$  is geometric.

Student A					Student B				
$n$	1	2	3	4	$n$	1	2	3	4
$t_n$	1	3	9	27	$t_n$	1	3	9	27
$\log_c(t_n)$	0	1	2	3	$\log_d(t_n)$	0	0.6826	1.3652	2.0478

Student B determined  $\log_d(t_n)$ , correct to the nearest ten thousandth.

**Written Response — 10%**

3. • Determine an expression for  $t_n$  and find the value of  $t_{10}$ .

- Determine the value of  $c$  in the expression  $\log_c(t_n)$ .

Written-response question 3 continues on the next page.



- Determine the value of  $d$ , correct to the nearest whole number, in the expression  $\log_d(t_n)$ .

- Upon examining the table, a student makes the following prediction. “Given the terms of a geometric sequence, the sequence of logarithms of each of these terms will produce a new sequence of values that is arithmetic.”

In order to prove this prediction, the student writes the general geometric sequence of  $n$  terms as  $a, ar, ar^2, \dots, ar^{n-1}$ .

The logarithm of each term of this geometric sequence produces the following sequence:

$$\log(a), \log(ar), \log(ar^2), \dots, \log(ar^{n-1})$$

Write a logical argument to prove that this sequence of logarithmic expressions is arithmetic.

*You have now completed the examination.  
If you have time, you may wish to check your answers.*



**Mathematics 30 Formula Sheet**

The following information may be useful in writing this examination.

- The roots of the quadratic equation  $ax^2 + bx + c = 0$  are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The distance between two points  $(x_1, y_1)$  and  $(x_2, y_2)$  is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

**Quadratic Relations**

- $e = \frac{|\overline{PF}|}{|\overline{PD}|}$

**Trigonometry**

- arc length  $a = r\theta$

- $\sin^2 A + \cos^2 A = 1$

- $1 + \tan^2 A = \sec^2 A$

- $1 + \cot^2 A = \csc^2 A$

- $\sin(A + B) = \sin A \cos B + \cos A \sin B$

- $\sin(A - B) = \sin A \cos B - \cos A \sin B$

- $\csc A = \frac{1}{\sin A}$

- $\sec A = \frac{1}{\cos A}$

- $\cot A = \frac{\cos A}{\sin A}$

- $\cos(A + B) = \cos A \cos B - \sin A \sin B$

- $\cos(A - B) = \cos A \cos B + \sin A \sin B$

**Permutations and Combinations**

- ${}_nP_r = \frac{n!}{(n-r)!}$

- ${}_nC_r = \frac{n!}{r!(n-r)!}$

- In the expansion of  $(x + y)^n$ , the general term is  $t_{k+1} = {}_nC_k x^{n-k} y^k$

**Sequences and Series**

- $t_n = a + (n-1)d$

- $S_n = \frac{n[2a + (n-1)d]}{2}$

- $S_n = n \left( \frac{a + t_n}{2} \right)$

- $t_n = ar^{n-1}$

- $S_n = \frac{a(r^n - 1)}{r - 1}, r \neq 1$

- $S_n = \frac{rt_n - a}{r - 1}, r \neq 1$

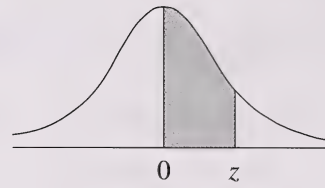
**Exponential and Logarithmic Functions**

- $\log_a mn = \log_a m + \log_a n$

- $\log_a \frac{m}{n} = \log_a m - \log_a n$

- $\log_a m^n = n \log_a m$

$$z = \frac{x - \mu}{\sigma}$$



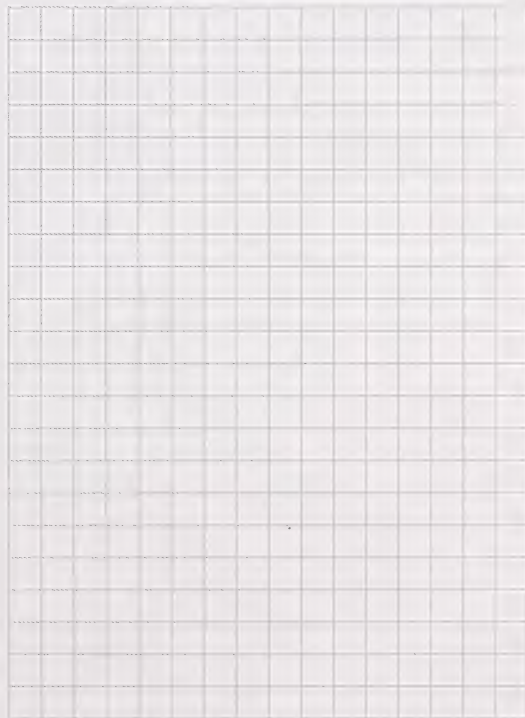
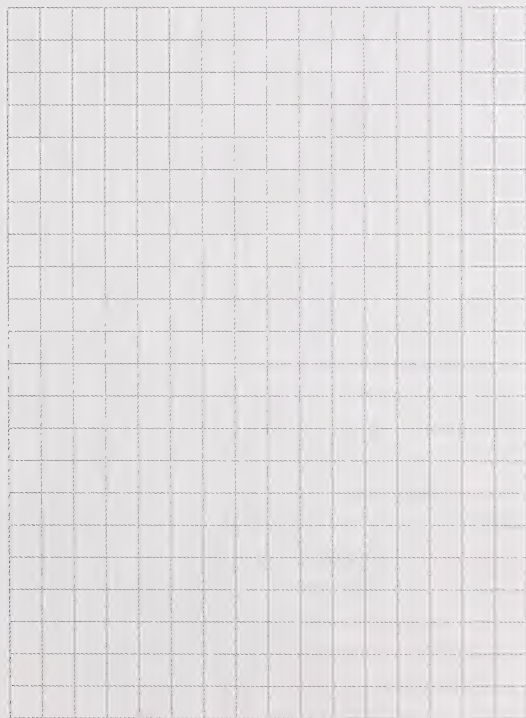
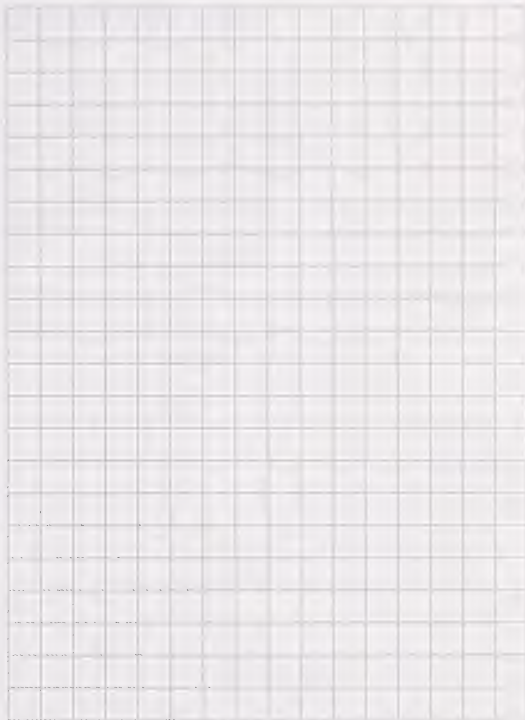
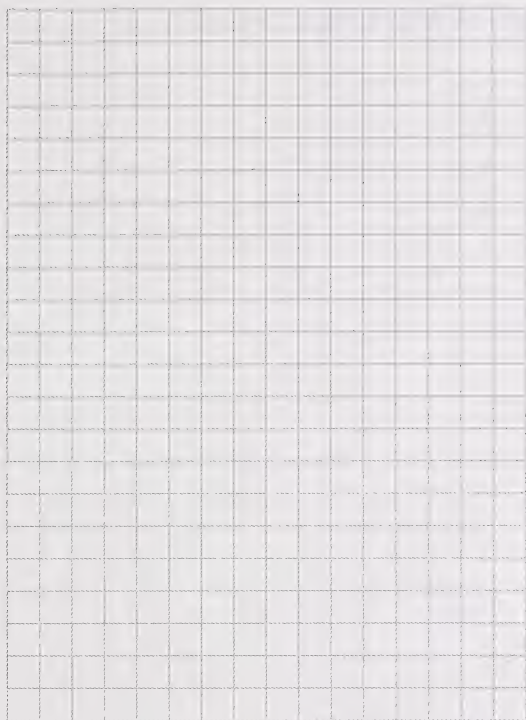
**Areas under the Standard Normal Curve**

<i>z</i>	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

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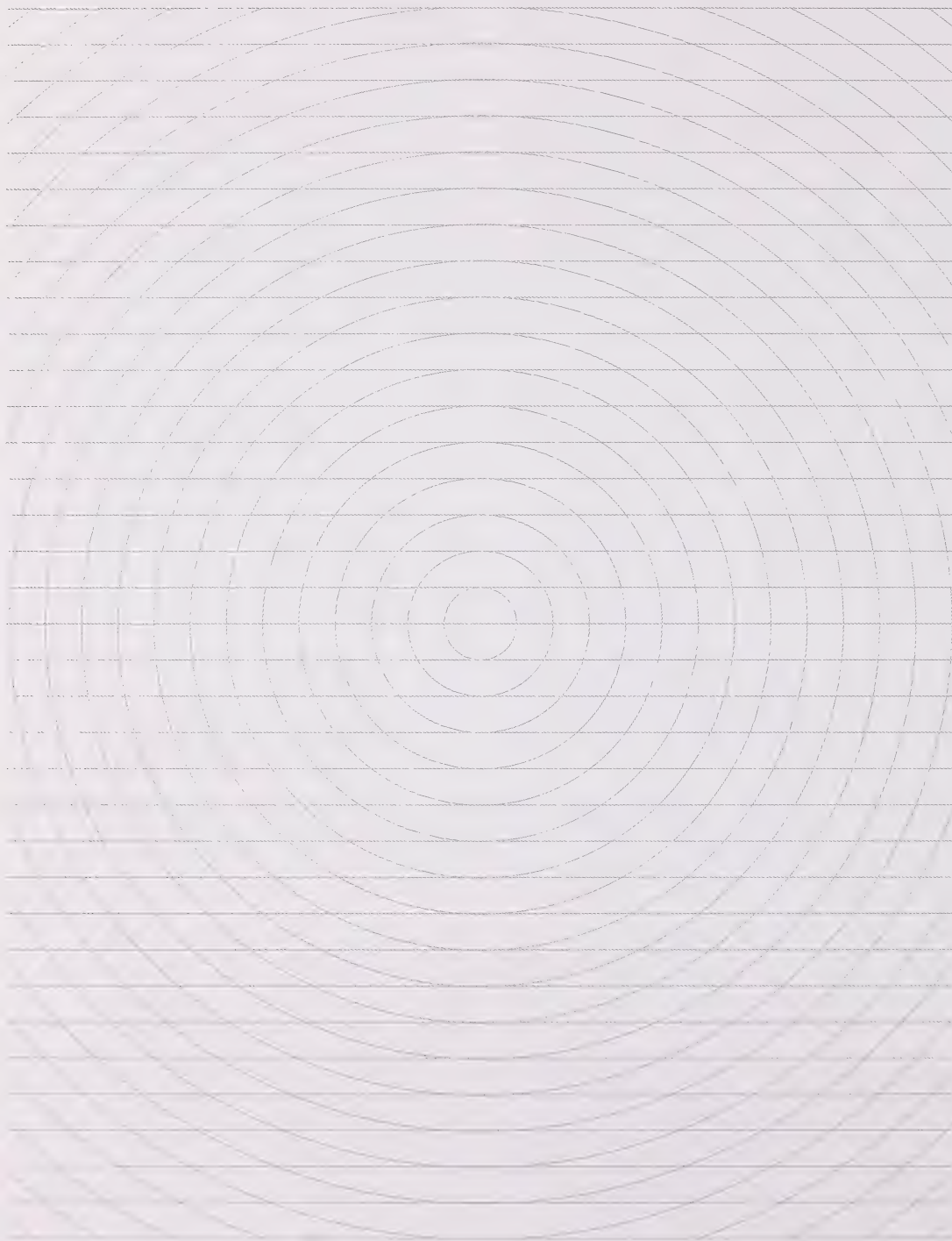
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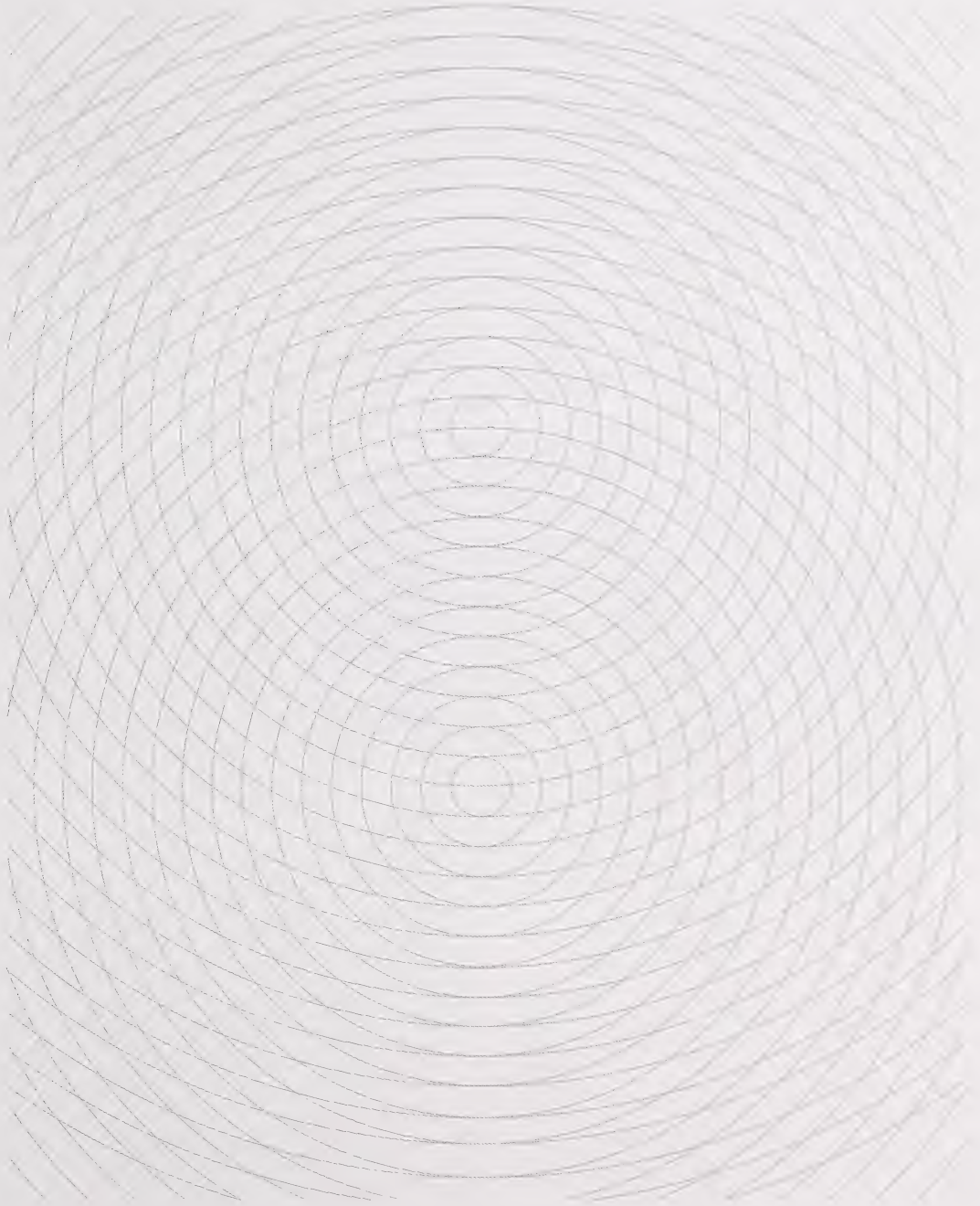
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